OPTIMAL DESIGN OF OXYGEN PRODUCTION SYSTEM BY PATTREN SEARCH METHODS

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ABSTRACT: Optimal design of Oxygen production system along with production rate, pressure in storage tank, compressor power and storage tank volume constraints were formulated in this study. In formulated optimization Oxygen production system the constraints were handled by using the exterior penalty functions. The derivative free methods were used for the optimization of this formulated problem. The methods were basically designed for unconstrained optimization problems. The optimum results of the Oxygen production optimization model were obtained by using MATLAB programming environment which demonstrated the effectiveness and applicability of the model. It was observed that the results of Nelder-Mead method were better than Hooke-Jeeves method. Nelder-Mead method was more efficient with respect to its function value and its number of function evaluations.

Keywords: Derivative free methods, penalty function, Oxygen production system, deterministic model and unconstrained optimization.

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INTRODUCTION

The optimization problems arise in almost all areas of real life like manufacturing, scheduling, business and engineering. By using optimization techniques the best solutions of the problems are obtained by utilizing minimum amount of limited resources (Ronald, 2002).

Optimization techniques are mainly divided into two categories i.e. Derivative Based Methods (DBMs) and Derivative Free Methods (DFMs) which are being frequently used in practical optimization (Tabassum *et al*., 2015). In this study the focus is on two direct search methods, namely, Nelder-Mead (NM) method and Hooke-Jeeves (HJ) method as has been reported by (Edger *et al*., 1988, William, 2001, Arora, 2004 and Isaac and Makoto, 2010). These methods are basically designed for solving un-constrained optimization problems. However these methods can also be applied to constrained optimization problems by changing them into unconstrained optimization problems. A traditional way for this purpose is the use of penalty function approach. Penalty functions depend on degree of constraint violation and the penalty factor which raises the objective function value for every infeasible solution and lowers the penalized objective function value for every solution nearer to the feasible region (Deb, 2003).

In the past when the derivatives of functions were taxing to calculate, the direct search methods were popular, but recently, the researchers have developed numerous tools for robust and automatic differentiation as well as modeling languages that compute derivatives automatically (Lagarias *et al*., 1998 and Price *et al*., 2002). In spite of all this, direct search method has its own importance. Particularly the maturation of simulation-based optimization has made it difficult to use derivative based method. Moreover, DBM cannot be applied to the problems in which the objective functions are not numeric in nature. An example of such problems is like optimal configuration of N-Queens on a square chess-board problem (Ali *el al*., 2015). The researchers have proposed a verity of DFM for diverse problems of practical optimization (Hooke and Jeeves, 1961, Joines and Houck, 1994 and Coello, 1999).

To change the constrained optimization into un-constrained one, by adding or subtracting the values from the objective functions is reported by (Deb, 2005).

In this script a mathematical model of Oxygen production system for minimum cost is reformulated and selected as a test case for the capabilities of NM and HJ methods. So far no such applications of these methods to such a challenging engineering optimization problem have been found. For selecting the best method there is a necessity to conduct comparative studies of their potential applications to modern world problems, like the one formulated in this study.

MATERIALS AND METHODS

The motivation for this research was to modify Oxygen production model. The derivative free methods were used for the optimization of Oxygen production model. These methods were basically designed for unconstrained optimization problems. In formulated optimization Oxygen production model the constraints were handled by using exterior penalty functions.

Hooke-Jeeves Method: For an N-dimensional problem HJ method required an initial point x_0, a set of N linearly independent search directions v_i, step-length parameters $\delta_i > 0$ and a parameter $\mu > 1$. The method used two types of moves given below:

Exploratory Move: This move was made on the current point by investigation along each direction according to the following formula:

$$x_{new} = x_0 \pm \delta_i v_i \quad \text{for all } i = 1, 2, 3, \ldots, N.$$

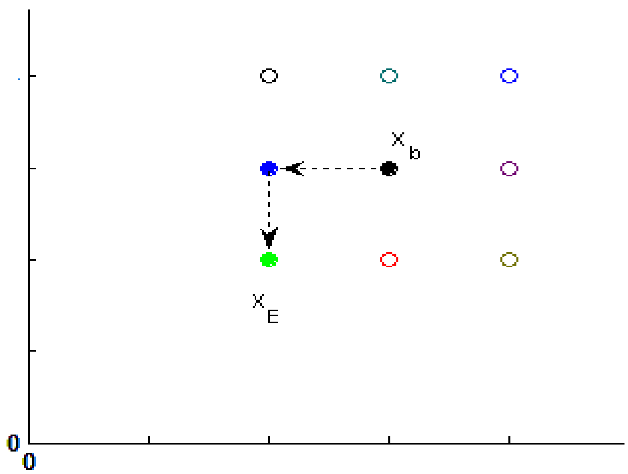


Fig-1. Successful exploratory move

Pattern Move: When exploratory move was completed and was accomplished successfully then pattern move was executed, by jumping from present base point along with a direction connecting and a new point was found. Once a pattern move was established it was possible to move as much as allowed. An enlargement parameter η, η ≥ 1, was used for this purpose. The pattern direction was found by the formula applied as $\underline{d} = z_E - z_b$. Therefore the new point, through pattern move, was found as given below

$$\underline{y}_b = z_E + \eta \underline{d} = z_E + \eta (z_E - z_b).$$

Nelder-Mead Simplex Method: While considering the initial simplex with three initial points i.e. y^0 = *Best Point*, y^1 = *Good Point*, y^2 = *Worst Point*. Take the centroid y^C of best and good points. Reflect the worst point through centroid, the y^r becomes the new point, which having equidistance from y^C to y^2. In this method there were several operations to be performed. Reflection occurred when $y^1 \geq y^r > y^0$.

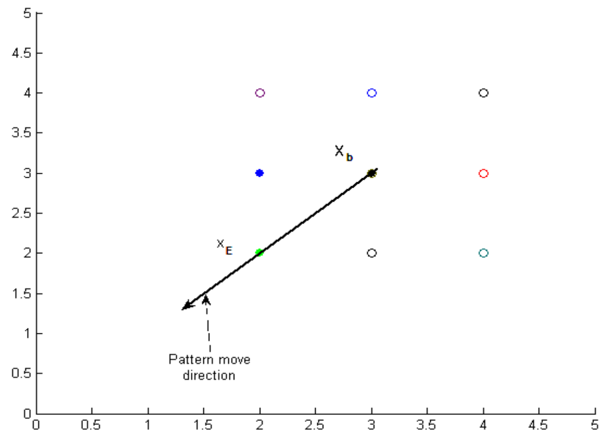


Fig-2: Pattern move direction

Mathematically, the reflected point y^r was given as

$$y^r = y^C + \delta^R (y^c - y^n)$$

and expansion occurred when $y^1 \geq y^0 > y^e$.

Mathematically, the expanded point y^e was given as

$$y^e = y^C + \delta^e (y^c - y^n)$$

In contraction when reflection point lies between the good and best vertex and it was generated two types. Outside contraction occurred when $y^2 \geq y^r > y^1$.

Mathematically, the expanded point y^{OC} was given as $y^{OC} = y^C + \delta^{OC}(y^c - y^n)$

Inside contraction occurred when $y^r \geq y^2$. Mathematically, the expanded point y^{iC} was given as $y^{IC} = y^C + \delta^{iC}(y^c - y^n)$. If no one from the above condition was satisfied then shrink was produced.

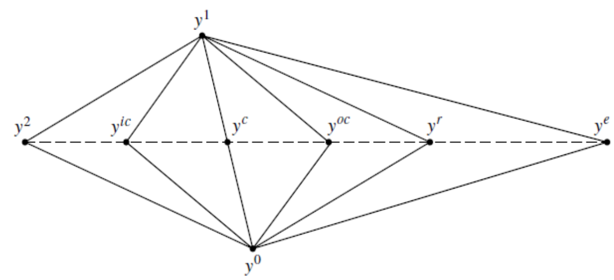
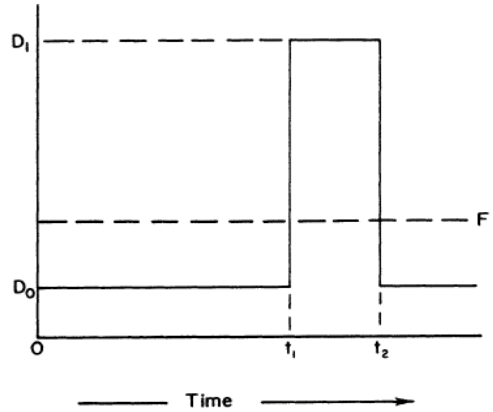


Fig-3: Steps of Nelder–Mead method

Oxygen production system: In this problem the prime objective was to minimize the cost of oxygen furnace. This oxygen furnace was used in chemical reactor for the supply of pure oxygen. Oxygen production system (Ravindran *et al*, 2006) contained oxygen plant, compressor and storage tank for oxygen furnace. Different kinds of variables were assigned and different kinds of constraints were generated, therefore the oxygen demanded varied with respect to time interval shown in Figure 4. Here t_1 was time interval for rate of low demand

D_0 and t_2 - t_1 time for rate of high demand D_1. Oxygen plants were designed to provide oxygen at a fixed rate.



$$D = \begin{Bmatrix} D_o & for & O \leq t \leq t_1 \\ D_1 & for & t_1 \leq t \leq t_2 \end{Bmatrix}$$

Fig-4: Cycle of demand

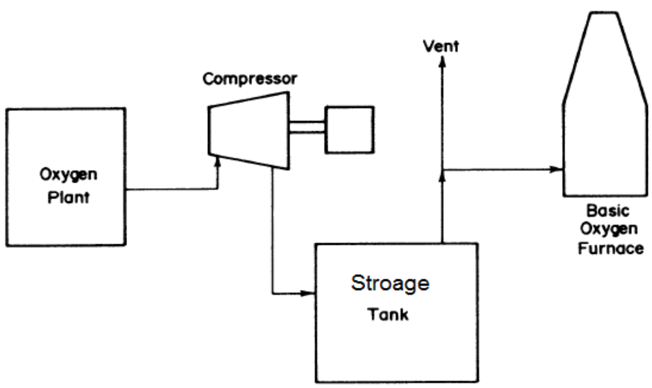


Fig-5: Oxygen production design

The capacity of oxygen plant = D_1.

Assumptions: Oxygen furnace and demand cycle were fixed, no external factors were imposed, storage tank had standard design and compression of ideal gas was isothermal.

Total annual cost = oxygen production cost + compressor operating cost + compressor cost + storage vessel

The model consisted on design equations that narrated independent variables (Bett *et al*, 1975).

Independent variables: Oxygen plant production rate F,

The compressor H,

Storage tank design capacities V,

The maximum tank pressure p.

I_{max} = maximum stored oxygen

By using law of corrected gas as $V = \frac{I_{max}}{M}\frac{RT}{p}z$

where R = *gas constant,*

T = *gas temperature,*

z = *compressibility factor,*

M = *molecular weight of Oxygen.*

From Figure 4, maximum oxygen = area under the demand curve between t_1 and t_2 and D_1 and F. Thus,

$I_{max} = (D_1 - F)(t_2 - t_1)$ (1)

Put the value I_{max} in above equation

$V = \frac{(D_1 - F)(t_2 - t_1)}{M} \frac{RT}{p} z$ (2)

As we now that the gas flow rate = $\frac{(D_1 - F)(t_2 - t_1)}{t_1}$

$H = \frac{(D_1 - F)(t_2 - t_1)}{t_1} \frac{RT}{k_1 k_2} ln\left(\frac{p}{\rho_0}\right)$ (3)

where k_1 = unit conversion factor,

k_2 = compressor efficiency,

ρ_0 = oxygen delivery pressure.

Rate of Oxygen plant F was sufficient, to supply the total demand of oxygen

$\geq \frac{(D_0 t_1) + D_1(t_2 - t_1)}{t_2}$ (4)

Maximum pressure of tank > delivered Oxygen pressure

$p \geq \rho_0$ (5)

Oxygen plant annual cost was $C_1\left(\frac{Rs}{year}\right) = a_1 + a_2 F$ (6)

where a_1 and a_2 were empirical constants.

empirical constants = (fuel+ water+ labor) costs for plants.

The capital cost for storage vessels

By using power correlation law, as $C_2(Rs) = b_1 V^{b_2}$ 7)

where b_1 and b_2 were empirical constants.

Similarly capital compressors cost attained from a correlation was

$C_3(Rs) = b_3 H^{b_4}$ (8)

Whereas compressor power cost was approximately = $b_5 t_1 H$

where b_5 was the power cost.

Total cost function

$Annual\ cost = a_1 + a_2 F + d(b_1 V^{b_2} + b_3 H^{b_4}) + N b_5 t_1 H$ (9)

where N = number of cycles per year

d =annual cost factor.

To minimize equation (9) represented complete design optimization problem that contained a suitable value of F, V, H, and p, cycle parameters were *(N, D_0, D_1, t_1, and t_2)*, cost parameters were *(a_1, a_2, b_1 to b_5, and d)* and physical parameters were *(T, ρ_0, k_2, z and M)* (Jen *et al*, 1968).

By using the new variables:

z_1= production rate of oxygen plant,

z_2= pressure in storage tank,

z_3= compressor power and

z_4= storage tank volume.

Non- linear programming model of oxygen design problem was as under

$$Minimizing\ Z = a_1 + a_2 z_1 + d(b_1 z_3^{b_2} + b_3 z_4^{b_4}) + N b_5 t_1 z_3$$

Subject to

$$t_2 z_1 \geq (D_0 t_1) + D_1(t_2 - t_1)$$

$$z_2 \geq \rho_0$$

$$z_3 = \frac{(D_1 - z_1)(t_2 - t_1)}{t_1} \frac{RT}{k_1 k_2} ln\left(\frac{p}{\rho_0}\right)$$

$$z_4 = \frac{RTz}{M} \frac{(D_1 - z_1)(t_2 - t_1)}{z_2}$$

$z_1, z_2, z_3, z_4 \geq 0$

Table 1. Different parameters for Oxygen supply system

Cycle Parameters	Value of Cost Parameters	Physical Parameters
N = 1	a_1= 61.8	T= 20^o
D_o = 2.5	a_2= 5.72	$\rho_0 = 200$
D_1 = 40	b_1= 0.0175	k_2= 0.5
t_1 = 0.6	b_2= 0.85	M=31.9999
t_2 = 2.5	b_3= 0.0094	$z = 28.2795$
	b_4= 0.75	
	b_5= 0.006	
	d= 1	

The final non-linear programming model of oxygen design problem became as

$$Minimizing\ Z = 61.8 + 5.72z_1 + (0.0175{z_3}^{0.85} + 0.0094{z_4}^{0.75}) + 0.0036z_3$$

Subject to

$z_1 \geq 17.5$

$z_2 \geq 200$

$$z_3 = 36.25\ \frac{(40-z_1)(0.4)}{0.6} ln\left(\frac{z_2}{200}\right)$$

$$z_4 = 348300 \frac{(40 - z_1)(0.4)}{z_2}$$

$z_1, z_2, z_3, z_4 \geq 0$

By using penalty function (Deb, 2005)

P(x) = r (max [0, g $_1$(x), g $_2$(x), g $_3$(x), g $_4$(x)])

$$Minimizing\ Z = 61.8 + 5.72z_1 + (0.0175{z_3}^{0.85} + 0.0094{z_4}^{0.75}) + 0.0036z_3 + 100\ [\max\ (0,\ [z_1 - 17.5], [z_2 - 200],\ [z_3 - 36.25\ \frac{(40-z_1)(0.4)}{0.6} ln\left(\frac{z_2}{200}\right)],\ [z_4 - 348300 \frac{(40-z_1)(0.4)}{z_2}])]$$

RESULTS AND DISCUSSION

Direct search methods had been popular because of their simplicity, flexibility, and reliability (Lewis *et al*, 2000). These methods had been shown to satisfy the first-order necessary conditions for a minimizer i.e., convergence to a stationary point (Lucidi and Mciandrone, 2002). It seemed remarkable that the given direct search methods neither required explicit derivative nor estimated derivative information. In most of the direct search methods a set of directions that span the search space was sufficient information to investigate the local behavior of the function (Rios and Sahinidis, 2012). To reduce the step length safely the set of directions had been queried (Nelder and Mead, 1965).

As per study conducted by (Hellinckx and Rijckeart, 1972 and Jen *et al*, 1968) have reported the solution of the above formulated problem with different setting of parameters. The Oxygen production system was solved by using geometric programming approach considering smaller values of the parameters. The best solution of the problem also gave the minimum cost of \$173.76 (Hellinckx and Rijckeart, 1972). The same problem was solved by using gradient based method with a minimum cost of 173.83\$ as reported by Jen *et al*, (1968). In this study the problem was solved by using two derivative free methods. The comparisons of the solutions found in this study are presented in table-2.

Table 2. Comparison of results

Sr. No.	Power cost \$/(HP-HR)	Production Rate	Maximum Pressure	Minimum power cost		
				Jen	HJ	NM
1	0.0015	17.5	802.82062	172.21	172.11700	172.11705
2	0.003	17.5	658.19221	172.85	172.74737	172.74745
3	0.006	17.5	473.69271	173.83	173.74617	173.74621
4	0.009	17.5	361.23119	174.52	174.45393	174.45394
5	0.012	17.5	283.80233	175.95	174.91330	174.91335
6	0.018	17.500001	200.00000	175.07	175.06747	175.06741
7	0.024	17.500001	200	175.07	175.06	175.0601

The previous studies witness that NM method is comparatively a low computation cost method. On the other hand HJ method provides guaranteed convergence for a number of differentiable functions (Dimitri *et al*, 2000). But the present study shows a different picture of the methods. It was observed that the solutions which are shown in table 2 at serial number 2 found by HJ and NM methods were approximately 0.059% better than the solution of Jen which was the maximum percentage in the results. It was also observed that the solutions which were shown in table 2 at serial number 6, 7 found by HJ and NM methods were approximately 0.006% better than the solution of which was the minimum percentage in the results. These comparisons had witnessed that Determinists DSMs like HJ and NM methods were yet better choices for solving such exponential type optimization problems in engineering design but HJ method is more reliable.

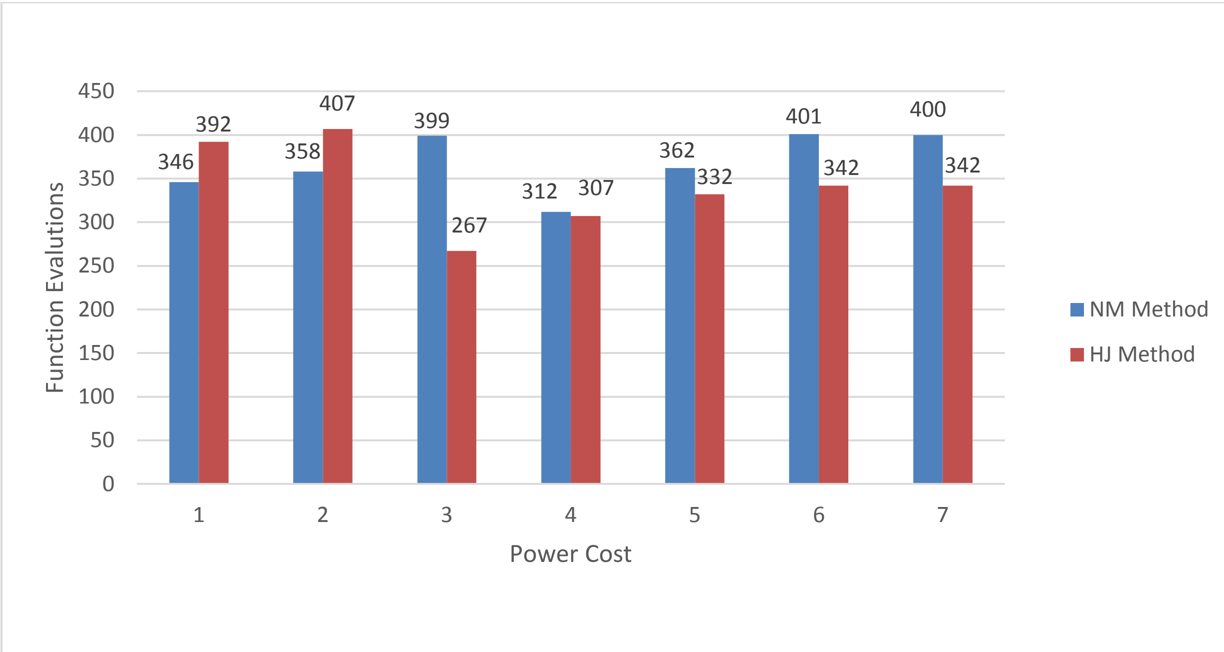


Fig-6. Comparison of function evaluation of NM and HJ methods

HJ method terminated when the step length fell below 10^{-9} and NM terminated when the maximum of 200×No of variables function evaluations were carried out. At these termination criteria the function evaluations by HJ method were smaller than those of NM method for the power cost rates (3-7) but were comparatively higher for (1-2). It was contradictory to the reported minimum computational cost of NM method. It was concluded that on the radical objective functions like the modeled one, HJ method may be a better and low cast choice.

Table 3. Comparison of Function Evaluations between NM Method and HJ Method.

Power cost	Function Evaluations	
	NM Method	HJ Method
0.0015	346	392
0.003	358	407
0.006	399	267
0.009	312	307
0.012	362	332
0.018	401	342
0.024	400	342

The above table also showed that when the power cost was small the number of function evaluations of NM method was less than that of HJ method and when the power cost increased gradually the performance of HJ method was getting better than NM method.

For optimum results of Oxygen design problem, a general-purpose solver was required. For numerical simulation of the oxygen design model, the programming environment of MATLAB was found quite supportive due to availability of a plenty of built-in functions. Another important advantage of MATLAB was the fact that parameters were easily settled for handling constraints.

Conclusion: The outcome performances of Hooke-Jeeves and Nelder-Mead methods experimented via a number of initial guesses were carried out on formulated Oxygen production system. It was concluded that performance of HJ method was promising with respect to its efficiency of solving such a problem with minimum computational efforts as compared to those of NM method. Through this work it was recommended that in any environment HJ method was a better choice as compared to the class of methods involving NM method.

REFERENCES

Ali J., M. Saeed, N. A. Chaudhry and M. F. Tabassum. (2015). New Mathematical Models of N-Queens Problem and its Solution by a Derivative-Free

Method of Optimization, Sci.Int., 27(3): 2005-2008

Arora, J.S. (2004). Introduction to Optimum Design. Amsterdam Boston Heidelberg; (USA) 277 p

Bett, K. E., Rowlinson J. S., and Saville G. (1975). Thermodynamics for Chemical Engineers. Cambridge MIT Press; (UK). 118 p

Coello, C.A.C. (1999). A Survey of Constraint Handling Techniques used with Evolutionary Algorithms. Technical report, lania-RI-99-04, laboratorio de Inform actica Avanzada, (Mexico).

Deb, K. (2006). An efficient constraint handling method for genetic algorithms. Comput Meth Appl Mech Eng. 18:311–318

Deb, K. (2003). Optimization for Engineering Desgin.. Rajkamal Electroic Press; (India). 96 p

Deb, K. (2005). Practical Optimization Using Evolutionary Methods. Kanpur Genetic Algorithms Laboratory. Report No. 2005008, (India).

Dimitri, P. N. Bertsekas and J. N. Tsitsiklis, (2000). Gradient Convergence in Gradient Methods with Errors. SIAM J. OPTIM. 10(3): 627–642.

Edger, Himmalblau, lesdon. (1988). Optimization of Chemical Processes. The McGraw-Hill Companies; (USA). 268 p

Hellinckx, L.J. and M.J. Rijckeart. (1972). Optimal Capacities of Production Facilities An Application of Geometric Programming. The Canadian journal of Chemical Engineering. 50: 148-150

Hooke, R. and T. A. Jeeves. (1961). Direct Search Solution of Numerical and Statistical problems. Journal of the Association for Computing Machinery; (USA). 8: 212-229

Isaac, E and O. Makoto. (2010). Optimization and Anti-Optimization of Structures under Uncertainty. Imperial College Press; London (UK). 211 p

Jen, F. C., C. C.Pegels and T. M. Dupuis. (1968). Optimal Capacities of Production Facilities. USA. Manag. Sci. 14:570–580

Joines, T. A. and C. R. Houck. (1994). On the Use of Non-stationary Penalty Functions to solve Non-linear Constrained Optimization Problems with GAs. In David Fogel, Proceedings of the first IEEE Conference of Evolutionary Computation; (Orlando) 579-584.

Lagarias, J. C., J. A.Reeds, M. H. Wright, and Wright P. E. (1998). Convergence Properties of the Nelder–Mead Simplex Method in Low Dimensions. SIAM J. Optim; 9:112–147

Lewis, R.M., V. Torczon, M. W. Trosset, (2000). Direct search methods: then and now. Journal of Computational and Applied Mathematics; 124(1): 191–207.

Lucidi, S. and M. Sciandrone, (2002). On the global convergence of derivative–free methods for unconstrained optimization. SIAM J. Opt; 13: 97–116

Nelder, J. A. and R. Mead. (1965). A simplex method for function minimization. Comput. J.; 7: 308–313

Price, C. J., I. D.Coope and Byatt D. (2002). A Convergent Variant of the Nelder-Mead algorithm. J. Optim. Theory Appl.; 113: 5–19.

Ravindran, A., K. M.Ragsdell and G. V. Reklaitis. (2006). Engineering Optimization Methods and Applications. John Wiley & Sons; (Canada). 10 p

Rios, L.M. and N. V. Sahinidis,(2012). Derivative-free optimization: a review of algorithms and comparison of software implementations. Journal of Global Optimization, 56(3):1247-1293

Ronald, L. R. (2002). Optimization in Operational Research. Pearson Education Press; (Singapore). 235 p

Tabassum, M.F., M.Saeed, Nazir Ahmad and A. Sana., (2015). Solution of War Planning Problem Using Derivative Free Methods. Sci.Int., 27(1): 395-398

William, D. N. (2001). Multidisciplinary Structural Design and Optimization for Performance, Cost, and Flexibility. University Press; California (USA). 139 p.