# ION-ACOUSTICBIPOLAR ELECTRIC FIELD SOLITARY STRUCTURESWITH TRAPPED ELECTRONS

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**ABSTRACT:** Bipolar electric field solitary (EFS) structures are essential components of space plasmas. Bipolar structures, which are associated with solitons, have been detected by numerous satellites across various areas of near-Earth plasmas, including the solar wind, Earth's magnetosphere, auroral zone, and Martian magnetosheath. A fluid model is presented in this paper that incorporates inertial warm ions and adiabatically trapped electrons, deriving the Sagdeev potential from fully nonlinear fluid equations. Our findings indicate that bipolar EFS structures, which correspond to compressive solitons, can emerge in such plasmas. The results of our model provide valuable insights for interpreting solitary structures in space plasmas where trapped electrons are present.

**Keywords:** Bipolar electric field structures; Trapped electrons; Generalized (r,q) distribution function; Sagdeev potential.

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## INTRODUCTION

Bipolar electric field solitary (EFS) structures represent a unique category of nonlinear waves that maintain shape while propagation. Such structures are frequently observed in the solar wind, Martian magnetosheath, reconnection sites within Earth's magnetosphere, Earth's bow shock, and the auroral region, as reported by various satellites (Thaller *et al*., 2022; Varghese *et al*., 2022; Vasko *et al*., 2020; Guo, 2014; Lapenta *et al*., 2011). In the auroral region, these structures typically arise when electrons are trapped in the magnetic field.

The study of one-dimensional solitary waves began with Washimi and Taniuti (1966), who employed the reductive perturbation method. Since then, numerous authors have extensively investigated nonlinear solitary waves under both small and finite amplitude conditions. The Sagdeev potential technique has proven to be the most effective method, as it comprises of fully nonlinear equations. Sagdeev was the first to analyze nonlinear ion-acoustic waves while considering the complete nonlinearity (Sagdeev, 1966). Concerning the Sagdeev potential, Witt and Lotko (1983) were pioneers in studying nonlinear ion-acoustic waves in Maxwellian plasma. Research on unmagnetized, multicomponent ion-acoustic waves has revealed only positive solitary structures (Lakhina *et al*., 2008). Additionally, studies on ion-acoustic waves in weakly relativistic plasmas with ion beams have shown that both rarefactive and compressive solitons can exist (Barman and Talukdar, 2012).

Bernstein, Greene, and Kruskal were the first to consider the trapping effect on the nonlinear dynamics of the wave caused by the wave itself. Later on, trapping was recognized as a microscopic phenomenon (Gurevich, 1967). Trapping was then experimentally confirmed as a microscopic phenomenon with 3/2 power nonlinearity, in contrast to the second order power nonlinearity observed when there is no trapping, as supported by numerical simulations (Sagdeev, 1966).The propagation features of ion-acoustic waves influenced by the trapping effect have been examined in both Maxwellian and non-Maxwellian plasmas (Abbasi *et al*., 1999; Mushtaq and Shah, 2006). These studies found that the propagation features were altered, with regular solitons observed in the former case and spiky solitons produced in the latter. Trapping has also been explored in one-dimensional cases within quantum plasmas (Shah *et al*., 2010), relativistic degenerate quantum plasmas (Shah *et al*., 2011), quantum plasmas with quantizing magnetic fields (Shah *et al*., 2012), and quantum dusty plasmas (Ayub *et al*., 2011).

Overall, it has been established that the transmission characteristics of nonlinear waves are modified when regular solitons are formed. Most previous studies on solitons have focused solely on ion dynamics or the nonthermal effects of electrons, or they have been limited to finite amplitude conditions. In this paper, we address this gap by considering adiabatically trapped electrons and analyzing fully nonlinear waves propagating in magnetized plasma through the derivation of the Sagdeev potential to obtain bipolar EFS structures.

**Theoretical Model:** In this study, we examine a plasma in which $B = B_0\hat{z}$ and is composed of inertial ions and trapped electrons. To characterize the underlying physics of such bipolar structures, we will employ following equations (Kalita *et al*., 1986; Kalita and Bhatta, 1994):

$$\frac{\partial n_i}{\partial \mathrm{t}} + \nabla.(n_i \boldsymbol{v_i}) = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{v}}{\partial \mathrm{t}} + (\boldsymbol{v_i}.\nabla)\boldsymbol{v_i} = -\frac{1}{n_i m_i}\boldsymbol{\nabla}\mathrm{p} - \frac{e}{m_i}\boldsymbol{\nabla}\varphi + \boldsymbol{v_i} \times \boldsymbol{\Omega}_{\mathrm{i}} \tag{2}$$

Let the propagation of the wave is considered to be in the *xz*-plane, the set of equations (1) and (2) can be written as:

$$\frac{\partial n_i}{\partial \mathrm{t}} + \frac{\partial}{\partial \mathrm{x}}(n_i v_{ix}) + \frac{\partial}{\partial \mathrm{z}}(n_i v_{iz}) = 0 \tag{3}$$

$$\frac{\partial v_{ix}}{\partial \mathrm{t}} + v_{ix}\frac{\partial v_{\mathrm{ix}}}{\partial \mathrm{x}} + v_{iz}\frac{\partial v_{\mathrm{iz}}}{\partial \mathrm{z}} = -\frac{\partial \varphi}{\partial \mathrm{x}} - \frac{\alpha}{n_i}\frac{\partial\, n_i}{\partial \mathrm{x}} + v_{\mathrm{iy}} \tag{4}$$

$$\frac{\partial v_{iy}}{\partial \mathrm{t}} + v_{ix}\frac{\partial v_{iy}}{\partial \mathrm{x}} + v_{iz}\frac{\partial v_{iy}}{\partial \mathrm{z}} = -v_{ix} \tag{5}$$

$$\frac{\partial v_z}{\partial \mathrm{t}} + v_{ix}\frac{\partial v_{iz}}{\partial \mathrm{x}} + v_{iz}\frac{\partial v_{iz}}{\partial \mathrm{z}} = -\frac{\partial \varphi}{\partial \mathrm{z}} - \frac{\alpha}{n_i}\frac{\partial\, n_i}{\partial \mathrm{z}} \tag{6}$$

In the equations presented above, $\alpha = T_i/T_e$ and the normalized quantities are defined as follows; $n$ by $n_0$,$\rho_i = c_s/\Omega_i$, (where $c_s = \sqrt{T_e/m_i}$ and $\Omega_i = e/(B_0 m_i)$),$t$ by$\Omega_i$,$v$ by $c$and $\varphi$by $T_e/e$.

Following the approach of Landau and Lifshitz (1980), trapped electrons are adhere to Maxwellian distribution. Consequently, the trapped electrons number density can be expressed as follows:

$$n_e = 1 + \varphi - \frac{4}{3\sqrt{\pi}}\varphi^{3/2} \tag{7}$$

It is important to note that power of 3/2 is introduced because of trapping in contrast to the typical second order terms observed when there is trapping.

By adopting a co-moving frame $\xi = k_x x + k_z z - Mt$, and applying the quasineutrality condition i.e., $n_i = n_e = n$we can reformulate equations (3)-(6) using equation (7) to express the three components of velocity, as

$$v_{ix} = \frac{M}{k_x}(n-1)\left[\frac{1}{n} - \frac{k_z^2}{2M^2}(1 + 2\alpha + n)\right] \tag{8}$$

$$v_{iy} = \frac{1}{k_x}\left[\left\{n(1 + \frac{2}{\sqrt{\pi}}\sqrt{\ln n}) + \alpha\right\}k_x^2 + (2\alpha + n)k_z^2 - \frac{2M^2}{n^2}\right]\frac{1}{n}\frac{d\,n}{d\xi} \tag{9}$$

$$v_{iz} = \frac{k_z}{2M}(n-1)(1 + 2\alpha + n) \tag{10}$$

Here$M = v/c_s$and$k_x^2 + k_y^2 = 1$. In deriving these equations, we apply boundary conditions$v_{ix} = v_{iz} = 0$ at $n = 1$as $\xi \longrightarrow \infty$.Upon solving these equations, we arrive at the following specific result

$$\frac{1}{2}\left(\frac{d\,n}{d\xi}\right)^2 + \Psi(n) = 0 \tag{11}$$

The conditions used in the derivation of above equation, is$\frac{dn}{d\xi} = 0$at $n = 1$. Here $\Psi(n)$ is referred to as the Sagdeev potential, or pseudo potential, is defined as follows:

$$\begin{aligned} &\Psi(n) \\ &= \frac{n^2}{\left[\left\{n(1 + \frac{2}{\sqrt{\pi}}\sqrt{\ln n}) + \alpha\right\}k_x^2 + (2\alpha + n)k_z^2 - \frac{2M^2}{n^2}\right]^2}\Bigg[(\frac{1}{36M^2n^2\sqrt{\pi}}(-36k_x^2n^3(-2M^2 \\ &+ k_z^2(1 + 2\alpha))\mathrm{DawsonF}[\sqrt{\mathrm{Log}[n]}] \\ &+ 2k_x^2n^3(-9\sqrt{2}n(m^2 - \mathrm{kz}^2\alpha)\mathrm{DawsonF}[\sqrt{2}\sqrt{\mathrm{Log}[n]}] \\ &+ 2\sqrt{3}k_z^2n^2\mathrm{DawsonF}[\sqrt{3}\sqrt{\mathrm{Log}[n]}] - 6(-3m^2(-2 + n) \\ &+ k_z^2(-3 + n^2 + 3(-2 + n)\alpha))\sqrt{\mathrm{Log}[n]}) \\ &+ 3\sqrt{\pi}(-2k_z^4n^3(-3 + n^2 + 6n\alpha + 6\alpha(-1 + 2\alpha)) \\ &+ 6m^2(m^2(-2 + 4n) + k_x^{2^2}n^3(-2 + n + 2\alpha)) \\ &+ k_z^2(\mathrm{kx}^2n^3(6 - 2n^2 - 9n\alpha - 12(-1 + \alpha)\alpha) + 6m^2(1 \\ &+ n^4 + 2\alpha - 4n\alpha + n^3(-2 + 4\alpha))) + 6n^2(-2k_x^2m^2\alpha \\ &+ 2\mathrm{kz}^4\alpha(1 + 2\alpha) + k_z^2(m^2(2 - 4\alpha) + k_x^2\alpha(1 \\ &+ 2\alpha)))\mathrm{Log}[n]))) \\ &- \frac{\{6m^2(2m^2 + k_x^2(-1 + 2\alpha)) - 2k_z^4(-2 + 6\alpha + 6\alpha(-1 + 2\alpha)) + k_z^2(12m^2\alpha + k_x^2(4 - 9\alpha - 12(-1 + \alpha)\alpha))\}}{12m^2}\Bigg] \end{aligned} \tag{12}$$

Solitary waves can be derived from equation (12) if the Sagdeev potential $\Psi(n)$meets following specific criteria

$$\begin{gathered} \Psi(N)|_{N=1} = \Psi(N)|_{N=N} = 0 \\ \Psi'(N)|_{N=1} = 0 \quad \Psi'(N)|_{N=N} > 0 \\ \text{and}\Psi''(N)|_{N=1} < 0 \end{gathered} \tag{13}$$

Therefore, to obtain solitary waves, the following condition must be satisfied

$$M^2 > (1 + \alpha)k_z^2 \tag{14}$$

**Numerical Results:** Numerical results are presented in this section when trapping of electrons are considered. Figure 1 illustrates Sagdeev potential structures for various Mach numbers. As the value of $M$ increases, Sagdeev potential's both depth and width increase. The associated soliton structure is depicted in Figure 2, where we observe that both the amplitude and width of the solitons increase with increasing $M$.

Figure 3 shows the Sagdeev potential structures for various directions of propagation, i.e. $k_z$. It is clear from the figure that with the increase in obliqueness, the width as well as depth decrease. Associated solitons are presented in Fig. 4, where the amplitude and width of solitons increase as obliqueness decreases. Figure 5 illustrates the Sagdeev potential structures for various ratios of $\alpha$. It is evident from this figure that with the increasing temperature ratio, Sagdeev potential's width and depth increase. Associated soliton are presented in Fig. 6, in which soliton's amplitude decreases but width increases as temperature ratio rises.

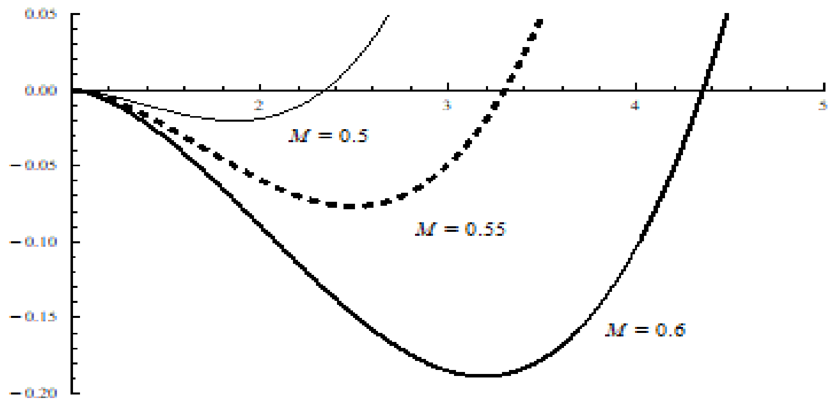


**Figure -1: Plots showing Sagdeev potential for different values of $M$ when $k_z = 0.4$and$\alpha = 0.1$.**

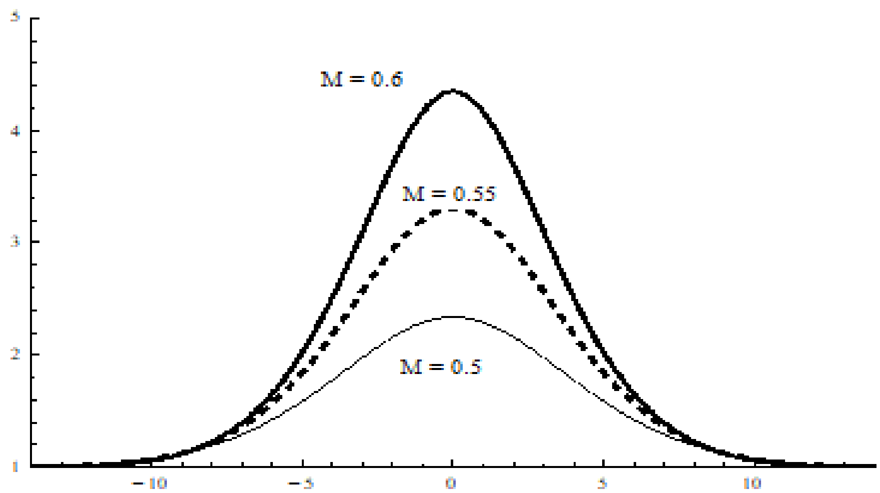


**Figure -2: Soliton structures corresponding to Fig.-1.**

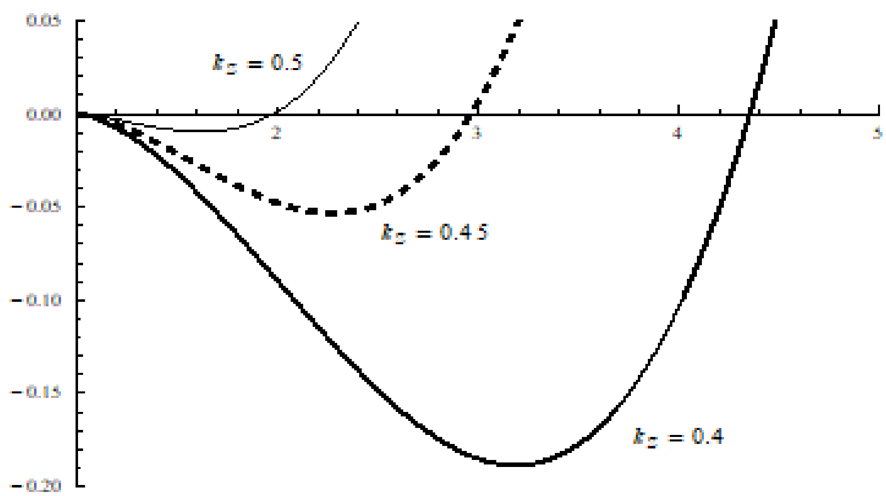


**Figure -3: Plots showing Sagdeev potential plots for different values of $k_z$ when $M = 0.4$and$\alpha = 0.1$.**

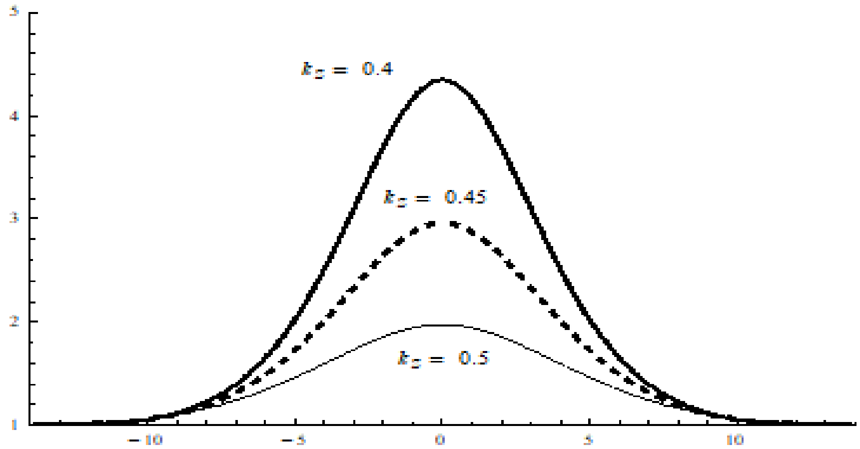


**Figure -4: Soliton structures corresponding to Fig.-3.**

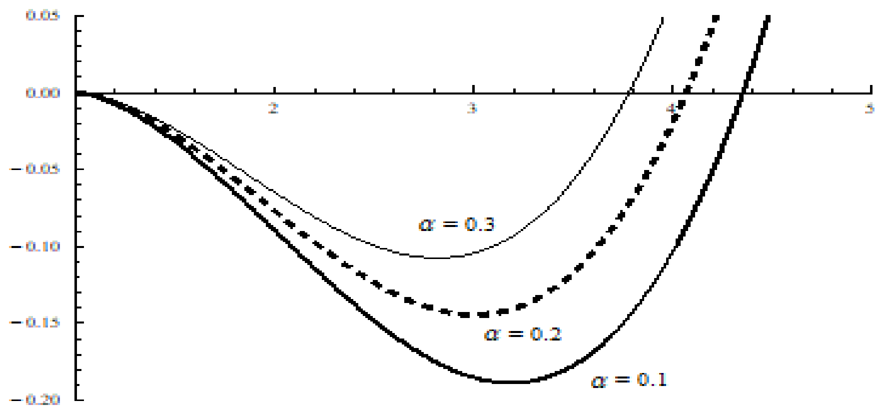


**Figure -5: Plots showing Sagdeev potential plots for different values of $\alpha$ when$k_z = 0.4$ and$M$= 0.4.**

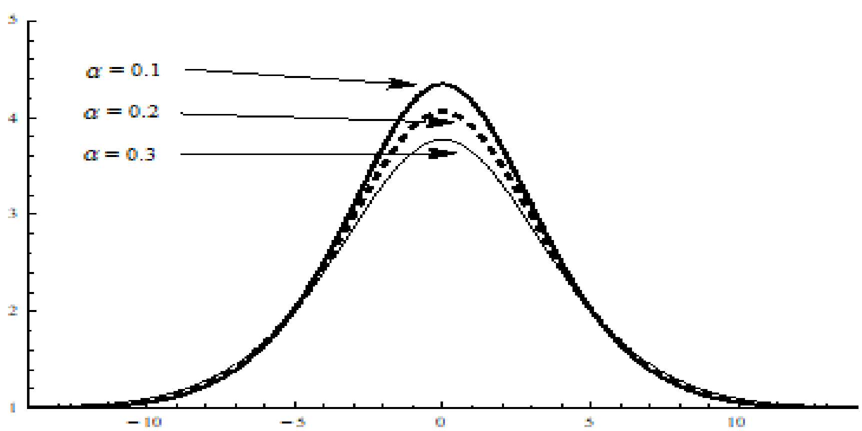


**Figure -6: Soliton structures corresponding to Fig.-5.**

**Bipolar EFS structures:** As $= -\frac{1}{n}\frac{d\,n}{d\xi}$ , so equation (11) takes the form

$$E = \mp\sqrt{-\frac{2}{n^2}\Psi(n)} \qquad (15)$$

By using Eq. (12) in above Eq. (15), the bipolar EFS structures can be obtained. Equation (15) is plotted numerically in Figs.(7)-(10) for various plasma parameters.

In Fig. 7, bipolar EFS are shown for different values of $M$, clearly indicating that with the increase in $M$, bipolar structures amplitude increases. Figure 8 presents bipolar EFS for varying values of $k_z$, demonstrating that the amplitude decreases as the parameter increases. Similarly, Fig.9 depicts bipolar EFS for different values of $\alpha$, revealing a decrease in amplitude with increasing values. Finally, Figure 10 illustrates bipolar EFS for various values of $v_e$, showing that the amplitude increases as the $v_e$ increases.

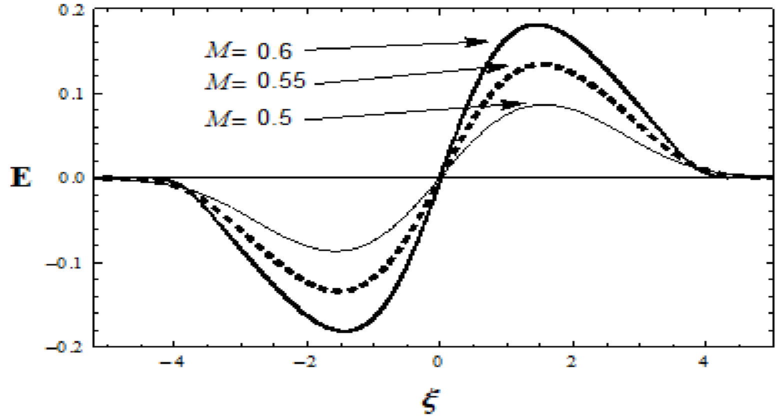


**Figure -7: Bipolar EFS structures for various Mach numbers when $\alpha = 0.1$,and $k_z = 0.4$.**

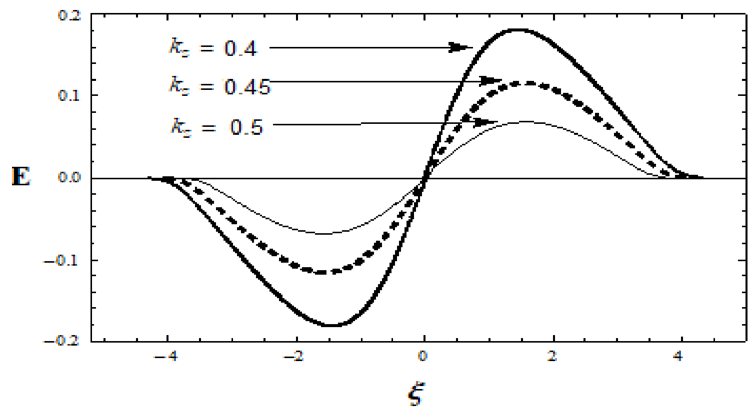


**Figure -8: Bipolar EFS structures for various $k_z$, when $\alpha = 0.1$ and $M = 0.4$.**

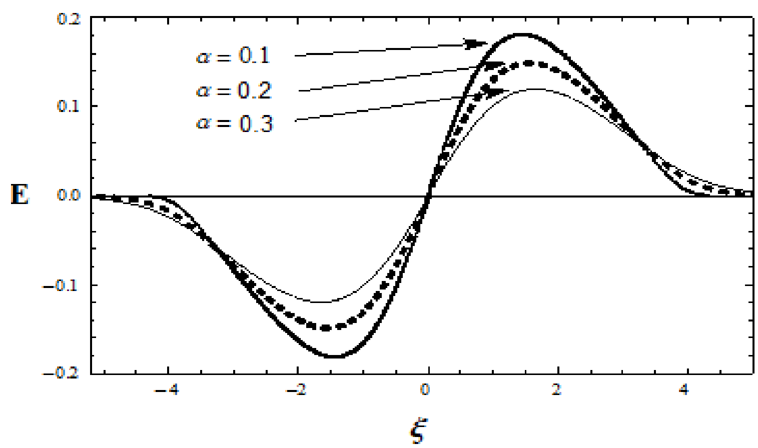


**Figure -9: Bipolar EFS structures for various$\alpha$, when $k_z = 0.4$and $M = 0.4$.**

**Summary and Conclusion:** Present manuscript, presented a nonlinear model developed to investigate the effect of trapped electrons on bipolar EFS structures. We derived the energy integral equation, so called the Sagdeev potential, by employing nonlinear fluid equations and numerically obtained associated bipolar EFS structures for various plasma parameters. Our model's findings are as follows: (i) both the width and amplitude of soliton increase with an increase in value of $M$; (ii) the soliton's amplitude and width decrease as obliqueness increases; (iii) when the ion-to-electron temperature ratio increases, the soliton's amplitude decreases while its width increases; (iv) the amplitude of bipolar electrostatic field structures rises with an increase in M; (v) the amplitude of these structures increases as the $\alpha$ decreases; and (vi) with the increase in the obliqueness, bipolar EFS structures decreases. We anticipate that the present model is valuable in interpreting space observations related to trapped electrons in the plasma.

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