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LINEAR DISPERSION RELATION OF KINETIC ALFVEN WAVES IN NONTHERMAL PLASMAS

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ABSTRACT: Alfven waves are important for energy transfer mechanisms and plasma heating in space and laboratory plasmas. Finite Larmor radius effect and oblique propagation induce parallel electric fields, making these waves dispersive. These waves' dispersive nature makes them key players for auroral acceleration, turbulent heating, magnetosphere-ionosphere coupling and energy deposition at rector edges. In this paper, we have investigated linear dynamics of kinetic Alfven waves (KAWs) in low-βelectron-ion plasmas by using two-potential theory and considering electrons following generalized (r, q) distribution function. Our results underscore the complex interplay between wavenumber, obliqueness, and distribution parameters in determining the frequency behavior of KAWs. The sensitivity of the fast mode to obliqueness and the significant influence of the r and q parameters highlight important factors that must be considered in understanding wave dynamics in plasma environments.

Keywords: Kinetic Alfven waves; Non-Maxellian distributions; Generalized (r,q) distribution; Space plasmas.

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INTRODUCTION

Alfven waves, the most fundamental magnetohydrodynamics (MHD) mode in plasma physics, have been researched extensively in the past decades. These waves are non-dispersive in nature due to a finite ion gyroradius effect. When the time scale is comparable to the ion orbital motion or the scale length is comparable to the ion kinetic scale, which is gyroradius of ions, these waves are capable of keeping a strong enough parallel electrostatic field along with magnetic fluctuations. Therefore, these waves can use transit time interactions or Landau damping to channelize wave-particle energy exchange. One plausible explanation for particle and energy transport is plasma turbulence(Gershman et al., 2017). An isotropic magnetic pressure $P_B = \frac{B_0^2}{2\mu_0}$and a magnetic tension $T_B = \frac{B_0^2}{\mu_0}$, along the magnetic field lines are the mechanical aspects of a magnetic fieldB_0. It is analogous to the theory of stretched strings, indicating that this tension may result in wave propagation transverse to the field lines.

When perpendicular wavelength is equal to the ion gyroradius, the electrons will remain attached to the field lines owing to their small Larmor radius, while the ions will have finite Larmor radius. This will cause charge separation, resulting in what are known as kinetic Alfven waves (KAWs). KAWs are far more intriguing than their customary MHD counterparts due to their dispersive nature. In the range of short perpendicular wavelengths($\lambda_\perp$) comparable to the ion gyroradius, kinetic Alfven waves (KAWs) are an extension of ion-acousticwaves(Cramer, 2001).The Mixture of KAW (when, $m_e/m_i \ll \beta \ll 1$) and inertial Alfvén waves (IAWs) (when, $\beta \ll m_e/m_i \ll 1$), β is thermal to magnetic pressure ratio, is known as dispersive Alfven waves (DAW) where, $\beta \equiv \frac{nT}{B_0^2/2\mu_0}$

Kinetic Alfven waves (KAWs) can be essential in the in-homogeneous heating of magnetoplasma structures because of their dispersive nature. The Linear theory of the kinetic Alfven wave has primarily developed around the different modified effects of the ideal Alfven wave caused by plasma processes, which render the MHD approach invalid and cause the Alfven wave to become dispersive.

In-situ measurements in space plasma reveal flat-tops and high-energy tails in distribution profile of charged particles species. In past, several distribution functions have been employed to tackle unexplained processes in space plasmas. The data investigation of space plasmas utilizing the generalized (r, q) distribution function showed better qualitative and quantitative agreement with the observed parameters than studies that employed kappa and other non-Maxwellian distribution functions(Qureshi et al., 2014, Qureshi et al., 2019). The generalized (r, q) distribution function not only encompass flat-topped distributions but also high energy tails and peaks at low energies in the distribution profile. In the limiting case, it reduce to kappa and Maxwellian distribution functions.

In this paper, we have developed Linear theory based on two-fluid model and investigated linear frequency of the waves where electrons are following generalized (r, q) distribution function.

Theoretical Investigations: In order to investigate the coupling interaction between ion acoustic waves (IAWs) and kinetic Alfven waves (KAWs) at low frequencies, we have accounted ion inertia effect and parallel current densities. Whereas, density of electrons is calculated using generalized (r, q) distribution function.

$$n_e = n_{eo}\left[1 + \alpha_1\Psi + \alpha_2\Psi^2\right] \quad (1)$$

Here α_1 and α_1 are constants depending upon generalized (r, q) distribution function,

$$\alpha_1 = \frac{(q-1)^{-\frac{1}{(1+r)}}\Gamma\left(\frac{1}{2(1+r)}\right)\Gamma\left(q-\frac{1}{2(1+r)}\right)}{2\,C\;\Gamma\left(\frac{3}{2(1+r)}\right)\Gamma\left(q-\frac{3}{2(1+r)}\right)} \quad (2)$$

$$\alpha_2 = \frac{3(q-1)^{-\frac{2}{(1+r)}}\Gamma\left(1-\frac{1}{2(1+r)}\right)\Gamma\left(q+\frac{1}{2(1+r)}\right)}{8\,C^2\;\Gamma\left(1+\frac{3}{2(1+r)}\right)\Gamma\left(q-\frac{3}{2(1+r)}\right)} \quad (3)$$

$$C = \frac{3\,(q-1)^{-1/(1+r)}\Gamma\left[q-\frac{3}{2+2\,r}\right]\Gamma\left[\frac{3}{2+2\,r}\right]}{2\,\Gamma\left[q-\frac{5}{2+2\,r}\right]\Gamma\left[\frac{5}{2+2\,r}\right]} \quad (4)$$

Here, r and q are spectral indices that characterize flat top and high-energy tail, respectively, in the distribution profile of particles under some conditions,$q > 1$ and $q(r+1) > 5/2$(Qureshi et al., 2013). In the limiting cases, when $r = 0$, $q \to \infty$ and $r = 0$, $q \to (\kappa + 1)$, the (r, q) distribution function reduces to the Maxwellian and kappa distributions respectively.

Propagation of wave is considered in the x-z plane and$B_z = B_0$ and $B_x = 0$. We made use of two-potential theory under the assumption $\frac{m_e}{m_i} < \beta < 1$.*i.e.*$E_x = \frac{-\partial\varphi}{\partial x}$,$E_z = \frac{-\partial\psi}{\partial z}$ and $E_y = 0$.

A set of governing equations is given below. The continuity equations for ion and electron are as follows

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_{ix}) + \frac{\partial}{\partial z}(n_i v_{iz}) = 0 \quad (5)$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial z}(n_e v_{ez}) = 0 \quad (6)$$

The momentum equation for ions is

$$m_i\left(\frac{\partial \boldsymbol{v_i}}{\partial t} + (\boldsymbol{v_i}.\boldsymbol{\nabla})\boldsymbol{v_i}\right) = e(\boldsymbol{E} + \boldsymbol{v_i} \times \boldsymbol{B}_o) \quad (7)$$

Using Ampere's law and induction equation(Masood et al., 2015), we obtain

$$\frac{\partial^4}{\partial x^2\partial z^2}(\phi - \psi) = \boldsymbol{\mu_o}\frac{\partial^2}{\partial t\,\partial z}j_z \quad (8)$$

where current density $j_z = n_e e(v_{iz} - v_{ez})$.

Using the above current density relation and Eq. (6), we get the following expression

$$\frac{\partial j_z}{\partial z} = e\frac{\partial n_e}{\partial t} + e\frac{\partial}{\partial z}(n_i v_{iz}) \quad (9)$$

Upon linearizing Eqs. (5)-(9) and simultaneously solving for linear frequency of coupled kinetic Alfven acoustic wave (CKAAW), we get the following linear dispersion relation (Sabeen et al., 2015)

$$\left(1 - \frac{V_A^2 k_z{}^2}{\omega^2}\right)\left(1 - \frac{c_s{}^2 k_z{}^2}{a_1\omega^2}\right) = \frac{V_A^2 k_z{}^2}{a\omega^2}\lambda_s \quad (10)$$

Here $V_A = \frac{B_o}{\mu_o n_i\, m_i}$, $\Omega_i = \frac{e\,B_o}{m_i}$ and $c_s = \sqrt{\frac{T_e}{m_i}}$ are Alfven velocity, ion-cyclotron frequency and sound speed respectively. Also $\lambda_s = k_x^2\rho_s^2$ is the coupling parameter and $\rho_s = \frac{c_s}{m_i}$is ion larmor radii. The term containing $a_1$on L. H. S of Eq.(10) (the acoustic term) is the ratio of sound velocity and phase velocity which becomes unity when phase velocity is much higher than sound velocity and we get the linear dispersion relation of KAWs(Hasegawa 1976, Khalid, et al., 2018)

$$\omega^2 = V_A{}^2 k_z{}^2\left(1 + \frac{\lambda_s}{a_1}\right)$$

RESULTS AND CONCLUSION

In this section, we present our graphical results of the linear dispersion relation for KAW.

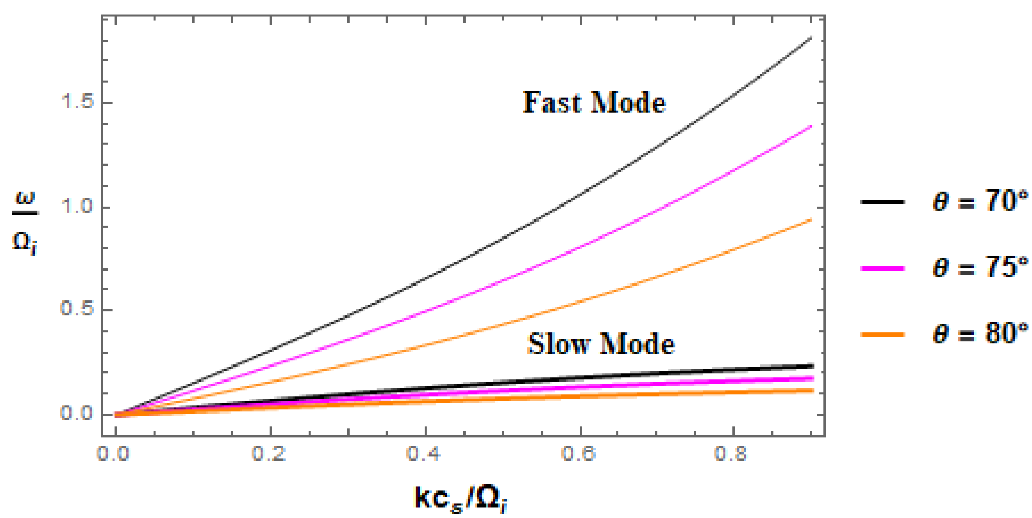

Figure-1: Plots of frequency vs. wave number of KAW when$\boldsymbol{\beta = 0.1}$.

Figure-1 shows the plot of the linear dispersion relation of KAW for different obliqueness. The graph depicts that the linear frequency has two modes: the Alfvenic mode (thick lines) and the acoustic mode (thin lines). The slow mode has been designated as the ion-acoustic branch, and the fast mode as the kinetic Alfven branch. It can be seen from the plot that the frequency of both fast and slow modes increases with the increase in wavenumber. Also, the frequency of both Alfvenic and acoustic modes decreases by increasing the obliqueness of the wave. However, in this case, the fast mode exhibits a more pronounced change in frequency.

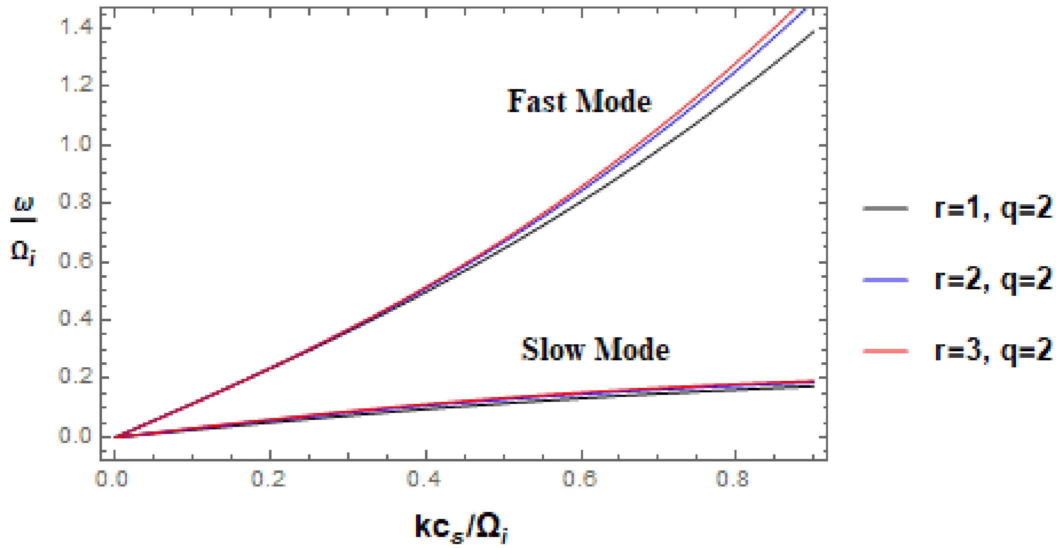

Figure-2:Plot of frequency vs. wave number for KAW when β=0.1.

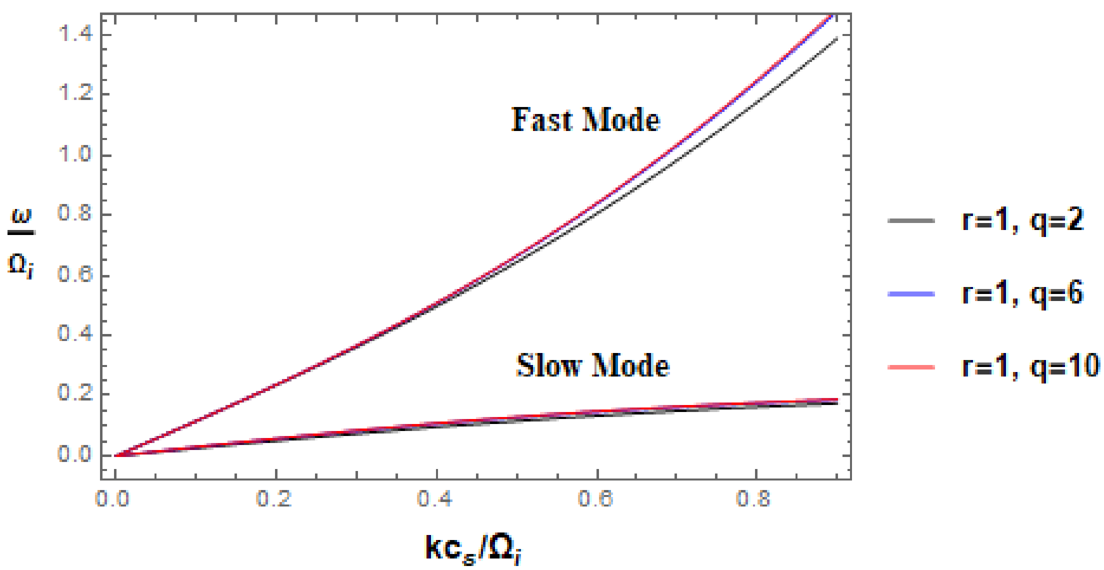

Figure-3:Plot of frequency vs. wave numberfor KAW when$\beta = 0.1$.

Figure-2 shows the plot of linear frequency of KAW for different values of $r$while keeping the q parameter fixed. The graph depicts that the linear frequency has two modes: the Alfvenic mode (thick lines) and the acoustic mode (thin lines). It is clear from the figure that the frequency of both Alfvenic mode and acoustic mode increases with the increase in the value of r.

Figure-3 depicts the plot of linear dispersion relation of KAW for different values of q, keeping the flatness parameter r value fixed. The graph depicts that the linear frequency has two modes: the Alfvenic mode (thick lines) and the acoustic mode (thin lines). The slow mode has been designated as the ion-acoustic branch, and the fast mode as the kinetic Alfven branch. It is clear from the figure that the linear frequency of both fast and slow modes increases with the increase in the value of q.

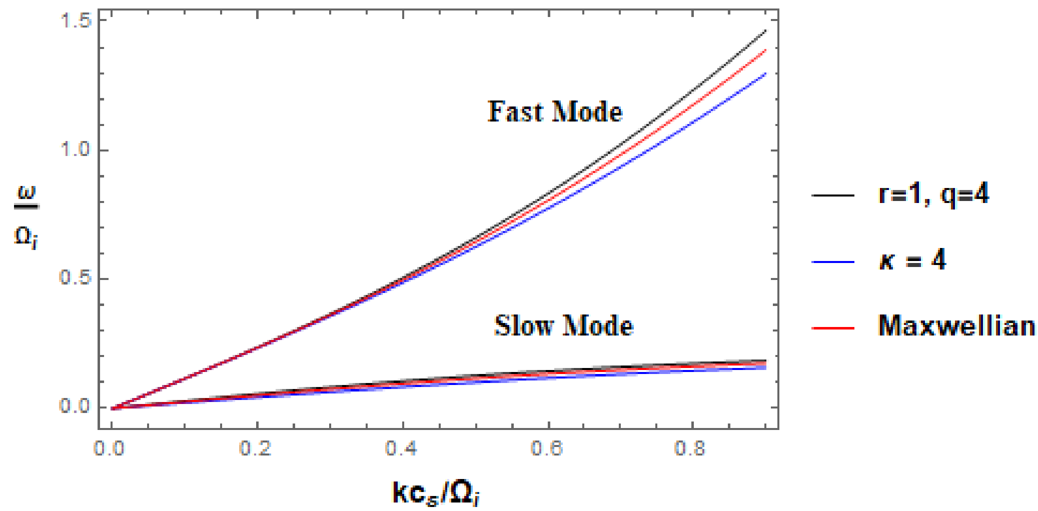

Figure-4: Plot of frequency vs. wave number for KAW when $\beta = 0.1$ and kappa = 4.

Figure 4 shows the comparison among kappa, Maxwellian, r and q values. It's obvious that frequency is highest for both Alfvenic and acoustic modes of the (r, q) distribution and lowest for the kappa distribution, frequency of Maxwellian distribution lies between (r, q) distribution and kappa distribution. On the other hand, the dispersion relations of various distributions differ for the kinetic Alfven branch, but only at higher values of the wave vector.

Summary and Conclusion: The dispersion relation consistently exhibits two different modes: the slow mode, corresponding to ion-acoustic branch, and the fast mode, corresponding to kinetic Alfvén branch. These modes are observed across all examined conditions, including variations in obliqueness, the parameter r, and the parameter q. Both the slow and fast modes experience an increase in frequency with the increase in wavenumber, indicating a direct correlation between wavenumber and wave frequency.

Increasing the obliqueness of the wave leads to a decrease in frequency for both modes. Notably, the fast mode shows a more significant reduction in frequency with increasing obliqueness, highlighting its sensitivity to the angle of propagation relative to the magnetic field. The frequency of both slow and fast modes increases with higher values of the parameter r while keeping q fixed. Similarly, increasing q while holding r constant also results in a rise in frequency for both modes.

These results suggest that both r and q parameters play crucial roles in modulating the frequency of KAWs, with higher values leading to higher wave frequencies.A comparative analysis among different distributions—kappa, Maxwellian, and the (r, q) distribution—reveals that the (r, q) distribution yields the highest real frequencies for both slow and fast modes. In contrast, the kappa distribution produces the lowest frequencies, with the Maxwellian distribution frequencies lying between the (r, q) and kappa distributions. The difference in dispersion relations across these distributions is more pronounced in the kinetic Alfvén branch, particularly at large values of the wave vector.

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